# kinematics Documentation <br> Release 0.1.7 

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This is the documentation for the kinematics package. It (will) provides a comprehensive attitude kinematics library to perform many common attitude transformations and computations.

## CHAPTER 1

## Installation Instructions

The kinematics package is simple to install. It is available from both pypi as well as anaconda. Furthermore, the only hard dependency is on the numpy package which is mostly likely already being used in your project.

## CHAPTER 2

## Installing

To install using pip, you can run:

```
pip install kinematics
```

Instead if you're already using the [Anaconda](www.anaconda.org) distribution, then it is preferrable to use conda to manage your installed packages.

This is beneficial as conda provides many additional tools to build independent environments and duplicate these environments between many systems. Here is a good blog post describing some of the approaches to utilizing pip and conda.

To instead install using conda simply use:

```
conda install -c skulumani kinematics
```


## CHAPTER 3

## Building from source

The package has been extensively tested on both OSX and Linux (Ubuntu). Binary distributions are provided which should also allow it to installed and used on Windows but this has not been tested.

To build from source, one should first clone the repository:

```
git clone https://github.com/skulumani/kinematics.git
```

Ensure that you have numpy installed on your system:

```
pip install numpy
```

or if you're using conda create a new enviornment with the appropriate dependencies:

```
conda create -n kinematics_env python=3 numpy
```

With the correct dependencies you can then install a development version of the package to ease development:

```
cd kinematics
pip setup.py -e .
```

chapter 5

Module Reference

## chapter 6

## attitude module

## Attitude Kinematics - transformations between attitude representations

This module provides a variety of functions to perform attitude kinematics. It transforms between a variety of attitude representations, provides functions for working with attitude,

## Example

Provide some examples of how to use this module

## References

[1] M. D. Shuster, A survey of attitude representations, Journal of the Astronautical Sciences, vol. 41, no. 8, pp. 439-517, 1993. [2] P. C. Hughes, Spacecraft Attitude Dynamics. Dover Publications, 2004. [3] J. R. Wertz, Spacecraft Attitude Determination and Control, vol. 73. Springer, 1978.

### 6.1 Requirements

List all the required components for this library to work properly.

```
kinematics.attitude.ang_veltoaxisangledot (angle, axis,Omega)
```

Compute kinematics for axis angle representation
kinematics.attitude.ang_veltodcmdot ( $R$, Omega)
Convert angular velocity to DCM dot Omega - angular velocity defined in body frame
kinematics.attitude.axisangledottoang_vel (angle, axis, angle_dot, axis_dot)
Convert axis angle represetnation to angular velocity in body frame
kinematics.attitude.axisangletodem (angle, axis)
kinematics.attitude.body313dot (theta, Omega)
kinematics.attitude.body313dot_to_ang_vel (theta, theta_dot)
kinematics.attitude.body313todem (theta)
kinematics.attitude.dcm2body313 (dcm)
Convert DCM to body Euler 3-1-3 angles
kinematics.attitude.dcmdottoang_vel ( $R, R d o t$ )
Convert a rotation matrix to angular velocity
w-angular velocity in inertial frame Omega - angular velocity in body frame
kinematics.attitude.dcmtoaxisangle ( $R$ )
kinematics.attitude.dcmtoquat (dcm)
Convert DCM to quaternion
This function will convert a rotation matrix, also called a direction cosine matrix into the equivalent quaternion.
dcm - $(\mathbf{3}, \mathbf{3})$ numpy array Numpy rotation matrix which defines a rotation from the $b$ to $a$ frame
quat - (4,) numpy array Array defining a quaterion where the quaternion is defined in terms of a vector and a scalar part. The vector is related to the eigen axis and equivalent in both reference frames [ x y z w ]

```
kinematics.attitude.hat_map(vec)
```

Return that hat map of a vector
Inputs: vec - 3 element vector
Outputs: skew - 3,3 skew symmetric matrix
kinematics.attitude.normalize (num_in, lower=0, upper $=360, b=$ False)
Normalize number to range [lower, upper) or [lower, upper].

## Parameters

- num (float) - The number to be normalized.
- lower (int) - Lower limit of range. Default is 0 .
- upper (int) - Upper limit of range. Default is 360 .
- $\mathbf{b}$ (bool) - Type of normalization. Default is False. See notes.

When $\mathrm{b}=$ True, the range must be symmetric about 0 . When $\mathrm{b}=$ False, the range must be symmetric about 0 or lower must be equal to 0 .

Returns $\mathbf{n}$ - A number in the range [lower, upper) or [lower, upper].
Return type float
Raises ValueError - If lower >= upper.

## Notes

If the keyword $b==$ False, then the normalization is done in the following way. Consider the numbers to be arranged in a circle, with the lower and upper ends sitting on top of each other. Moving past one limit, takes the number into the beginning of the other end. For example, if range is [ $0-360$ ), then 361 becomes 1 and 360 becomes 0 . Negative numbers move from higher to lower numbers. So, -1 normalized to [0-360) becomes 359 .

When $\mathrm{b}=$ False range must be symmetric about 0 or lower $=0$.

If the keyword $b==$ True, then the given number is considered to "bounce" between the two limits. So, -91 normalized to $[-90,90]$, becomes -89 , instead of 89 . In this case the range is [lower, upper]. This code is based on the function fmt_delta of TPM.

When $\mathrm{b}=$ True range must be symmetric about 0 .

## Examples

```
>>> normalize(-270,-180,180)
90.0
>>> import math
>>> math.degrees(normalize(-2\starmath.pi,-math.pi,math.pi))
0.0
>>> normalize(-180, -180, 180)
-180.0
>>> normalize(180, -180, 180)
-180.0
>>> normalize(180, -180, 180, b=True)
180.0
>>> normalize(181,-180,180)
-179.0
>>> normalize(181, -180, 180, b=True)
179.0
>>> normalize(-180,0,360)
180.0
>>> normalize(36,0,24)
12.0
>>> normalize(368.5,-180,180)
8.5
>>> normalize(-100, -90, 90)
80.0
>>> normalize(-100, -90, 90, b=True)
-80.0
>>> normalize(100, -90, 90, b=True)
80.0
>>> normalize(181, -90, 90, b=True)
-1.0
>>> normalize(270, -90, 90, b=True)
-90.0
>>> normalize(271, -90, 90, b=True)
-89.0
```

kinematics.attitude.quatdot (quat, Omega)
kinematics.attitude.quatdot_ang_vel (quat, quat_dot)
kinematics.attitude.quattodem (quat)
Convert quaternion to DCM
This function will convert a quaternion to the equivalent rotation matrix, or direction cosine matrix.
quat - (4,) numpy array Array defining a quaterion where the quaternion is defined in terms of a vector and a scalar part. The vector is related to the eigen axis and equivalent in both reference frames [ x y z w ]
dcm - $(\mathbf{3}, \mathbf{3})$ numpy array Numpy rotation matrix which defines a rotation from the $b$ to a frame
kinematics.attitude. rot1 (angle, form= $\mathbf{u}^{\prime} c^{\prime}$ )
Euler rotation about first axis

This computes the rotation matrix associated with a rotation about the first axis. It will output matrices assuming column or row format vectors.

For example, to transform a vector from reference frame $b$ to reference frame $a$ :
Column Vectors : $a=\operatorname{rot} 1$ (angle, ' $c$ '). $\operatorname{dot}(b)$ Row Vectors : $a=b . \operatorname{dot}\left(\operatorname{rot} 1\left(\right.\right.$ angle, ${ }^{\prime} r$ '))
It should be clear that $\operatorname{rot} 1$ (angle, ' $c$ ') $=\operatorname{rot} 1$ (angle, ${ }^{\prime} r$ ').T
Parameters

- angle (float) - Angle of rotation about first axis. In radians
- form (str) - Flag to choose between row or column vector convention.

Returns mat - Rotation matrix
Return type numpy.ndarray of shape $(3,3)$
kinematics.attitude. rot2 (angle, form=u'c')
Euler rotation about second axis
This computes the rotation matrix associated with a rotation about the first axis. It will output matrices assuming column or row format vectors.

For example, to transform a vector from reference frame $b$ to reference frame $a$ :
Column Vectors : $a=\operatorname{rot} 2($ angle, ' $c$ '). $\operatorname{dot}(b)$ Row Vectors : $a=b . \operatorname{dot}(\operatorname{rot} 1$ (angle, 'r'))
It should be clear that $\operatorname{rot} 2\left(\right.$ angle, $\quad \mathrm{c}$ ') $=\operatorname{rot} 2\left(\right.$ angle, ${ }^{\prime} r$ ').T

## Parameters

- angle (float) - Angle of rotation about second axis. In radians
- form (str) - Flag to choose between row or column vector convention.

Returns mat - Rotation matrix
Return type numpy.ndarray of shape $(3,3)$
kinematics.attitude. rot3 (angle, form=u'c')
Euler rotation about thrid axis
This computes the rotation matrix associated with a rotation about the third axis. It will output matrices assuming column or row format vectors.

For example, to transform a vector from reference frame $b$ to reference frame $a$ :
Column Vectors : $a=\operatorname{rot} 3($ angle, ' $c$ ') $\cdot \operatorname{dot}(b)$ Row Vectors : $a=b \cdot \operatorname{dot}($ rot $3($ angle, 'r'))
It should be clear that rot3(angle, ' $c$ ') $=\operatorname{rot} 3$ (angle, $\quad$ ' $r$ '). T

## Parameters

- angle (float) - Angle of rotation about first axis. In radians
- form (str) - Flag to choose between row or column vector convention.

Returns mat - Rotation matrix
Return type numpy.ndarray of shape $(3,3)$

```
kinematics.attitude.test_rot_mat_in_special_orthogonal_group ( }R\mathrm{ )
kinematics.attitude.unit_vector(q)
    Unit vector in direction of q
    qhat = unit_vector(q)
```

Parameters $\mathbf{q}((n$,$) numpy array) - Vector in Rn$
Returns qhat - The unit vector in the direction of $q$
Return type ( n, ) numpy array

## Notes

You may include some math:

$$
\hat{q}=\frac{q}{q}
$$

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```
kinematics.attitude.vee_map(skew)
```

Return the vee map of a vector

## chapter 7

## sphere module

n -Sphere operations
This module holds some useful functions for dealing with elements of $n$-spheres. Most usually we tend to deal with the 1 -sphere and the 2 -sphere.

```
kinematics.sphere.perturb_vec (q, cone_half_angle=2)
```

Perturb a vector randomly
$\mathrm{qp}=$ perturb_vec(q, cone_half_angle=2)

## Parameters

- $\mathbf{q}((n$,$) numpy array) - Vector to perturb$
- cone_half_angle (float) - Maximum angle to perturb the vector in degrees


## Returns

- perturbed (( $n$, ) numpy array) - Perturbed numpy array
- Author
-     - 
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## References

kinematics.sphere. rand (n, **kwargs)
Random vector from the $n$-Sphere
This function will return a random vector which is an element of the $n$-Sphere. The $n$-Sphere is defined as a vector in $\mathrm{R}^{\wedge} \mathrm{n}+1$ with a norm of one.

Basically, we'll find a random vector in $\mathrm{R}^{\wedge} \mathrm{n}+1$ and normalize it. This uses the method of Marsaglia 1972.

## Parameters None -

Returns Random ( $\mathrm{n}+1$, ) numpy vector with a norm of 1

Return type rvec
kinematics.sphere.tan_rand (q, seed=9)
Find a random vector in the tangent space of the n sphere
This function will find a random orthogonal vector to q .
Parameters $\mathbf{q}-(\mathrm{n}+1$,$) array which is in the \mathrm{n}$-sphere
Returns ( $\mathrm{n}+1$, ) array which is orthogonal to n -sphere and also random
Return type qd

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